MATH5010 Linear Analysis (2022-23): Homework 4. Deadline: 23 Oct 2022

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Suppose that the Euclidean space $\mathbb{R}^n$ is endowed with the usual norm, that is, $\|x\|_2 := \sqrt{\sum_{k=1}^n |x_k|^2}$ for $x = (x_1, ..., x_n) \in \mathbb{R}^n$. For each $x \in \mathbb{R}^n$, put $\|x\|_\infty := \max\limits_{1\le k\le n} |x_k|$. Using the definition of equivalent norms, show that the norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$ are equivalent on $\mathbb{R}^n$.

2. Let $X := \mathbb{R}^2$ be a two dimensional real vector space and let A be the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$. Define a mapping $T : X \to X$ by $Tx = Ax$ for $x \in X$. Suppose that X is endowed with the $\|\cdot\|_\infty$-norm, that is $\|x\|_\infty := \max(|x_1|, |x_2|)$ for $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in X$. Find $\|T\|$.

3. Recall that c_{00} denotes the finite sequence space which is equipped with the $\|\cdot\|_\infty$-norm. Let $T : c_{00} \to c_{00}$ be the linear map given by

$$T(x)(k) := kx(k)$$

for $k = 1, 2...$ and $x \in c_{00}$. Show that T is a discontinuous map.

$* * *$ **End** $* * *$